An effective-β vector for linear planetary waves on a weak mean flow

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Abstract

An effective-β vector and an accompanying mean-flow advection vector are computed for linear planetary waves on a weak, horizontally uniform mean flow, for arbitrary stratification and vertical shear profiles. These vectors arise from the projections of mean-flow vertical shear and horizontal advection terms onto vertical structure functions related to the rest-state linear vertical mode. As such, these vectors are independent of depth, and determine the propagation direction and speed for the linear planetary waves on the weak mean flow. The resulting dispersion relation gives the modified wave frequency as the sum of a Doppler shift term and a dispersion function that has the same form as the rest-state planetary-wave dispersion function, but relative to the effective-β vector. The propagation of long waves remains nondispersive. A modal decomposition of the mean flow illustrates the non-Doppler effect. For two examples of climatological mean-flow and stratification profiles, the theory is shown to reproduce accurately the modified long-wave zonal phase speeds from the full linear long-wave solution on a purely zonal mean flow.

1 The effective-β vector

Quasi-geostrophic linear planetary waves on a resting ocean satisfy a dispersion relation of the form,

$$\omega = \Omega(k,l) = -\frac{\beta k}{K^2 + \lambda_R^{-2}}, \tag{1.1}$$

where ω is frequency, k is zonal wavenumber, $K^2 = k^2 + l^2$ is total squared wavenumber, λ_R is a deformation radius, and β is the rate of change of the Coriolis parameter with latitude (Pedlosky, 1987). Such waves have purely westward phase propagation, with zonal phase speed $c = \omega/k < 0$. The scalar β can be interpreted as the magnitude of a β-vector that is equal to the vector gradient of the Coriolis parameter and directed northward; the westward phase propagation direction is then ninety degrees counterclockwise of this vector. In the presence of a mean flow, this dispersion relation and the associated propagation are altered. In the following, an effective-β vector for linear waves on a non-zero mean flow is computed that determines the analogous pseudo-westward phase propagation, generalizing an approach and result for small-amplitude waves on a zonal mean flow presented by Killworth et al. (1997).

For linear waves on a large-scale mean flow that varies only in the vertical, the quasi-geostrophic potential vorticity equation for streamfunction (scaled pressure) $\psi(x,y,z,t)$ is

$$\left[\frac{\partial}{\partial t} + U(z)\frac{\partial}{\partial x} + V(z)\frac{\partial}{\partial y}\right] q + \frac{\partial \psi}{\partial x}\left(\beta + \frac{\partial Q}{\partial y}\right) - \frac{\partial \psi}{\partial y}\frac{\partial Q}{\partial x} = 0, \tag{1.2}$$

where

$$q = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial \psi}{\partial z}\right), \tag{1.3}$$

is the disturbance potential vorticity, $\mathbf{U} = (U,V)$ is the mean geostrophic flow, and $\nabla Q =$

(Q_x, Q_y),

$$Q_x = \frac{d}{dz}\left(\frac{f^2}{N^2}\frac{dV}{dz}\right), \quad Q_y = -\frac{d}{dz}\left(\frac{f^2}{N^2}\frac{dU}{dz}\right), \tag{1.4}$$

is the mean-flow potential vorticity gradient (Pedlosky, 1987). Since the coefficients in (1.2) are independent of x, y, and t, let

$$\psi(x,y,z,t) = P(z)\exp[-i(kx+ly-\omega t)]. \tag{1.5}$$

Then $P(z)$ must satisfy the ordinary differential equation,

$$(\omega - kU - lV)\left[-K^2P + \frac{d}{dz}\left(\frac{f^2}{N^2}\frac{dP}{dz}\right)\right] - \left\{k\left[\beta - \frac{d}{dz}\left(\frac{f^2}{N^2}\frac{dU}{dz}\right)\right] - l\frac{d}{dz}\left(\frac{f^2}{N^2}\frac{dV}{dz}\right)\right\}P = 0, \tag{1.6}$$

were $K^2 = k^2 + l^2$. The equation (1.6) must be solved subject to the boundary conditions,

$$(\omega - kU - lV)\frac{dP}{dz} + (k\frac{dU}{dz} + l\frac{dV}{dz})P = 0 \quad \text{at} \quad z = \{-H, 0\}, \tag{1.7}$$

representing the inviscid flat-bottom and rigid-lid no-normal-flow conditions in the presence of the mean geostrophic flow $\mathbf{U}$.

Now, suppose that the mean flow is weak, so that the terms involving mean flow effects in (1.6) and (1.7) are small relative to the others. Thus, let

$$(U,V) = \varepsilon(\tilde{U},\tilde{V}), \quad \omega \approx \omega_0 + \varepsilon\omega_1, \quad P \approx P_0 + \varepsilon P_1, \tag{1.8}$$

in (1.6) and (1.7), with $\varepsilon << 1$ measuring the relative strength of the mean flow effects. At $O(1)$, the standard equation for vertical modes on an ocean at rest is obtained,

$$\frac{d}{dz}\left(\frac{f^2}{N^2}\frac{dP_0}{dz}\right) + \lambda^{-2}P_0 = 0, \quad \lambda^{-2} = -\frac{\beta k}{\omega_0} - K^2, \tag{1.9}$$

with the standard rest-state boundary condition

$$\frac{dP_0}{dz} = 0 \quad \text{at} \quad z = \{-H, 0\}. \tag{1.10}$$

This is an eigenvalue problem for λ. For general $N(z)$, the n-th rest-state vertical mode and eigenvalue are denoted by $P_0^{(n)}$ and λ_n, respectively, with $n = \{0, 1, 2, ...\}$, resulting in the dispersion relation (1.1), with $\omega_0 = \Omega(k,l)$ for $\lambda_R = \lambda_n$. The modes $P_0^{(n)}$ are orthogonal,

$$\int_{-H}^{0} P_0^{(n)} P_0^{(p)}|_{p\neq n}\, dz = 0, \tag{1.11}$$

and may be normalized so that

$$\int_{-H}^{0} \left(P_0^{(n)}\right)^2 dz = 1. \tag{1.12}$$

The focus here is on the first internal mode, so that $\lambda = \lambda_1$ and $P_0 = P_0^{(1)}$ in the following. In principle, the calculations go through in exactly the same way for higher rest-state modes; however, for the slower-moving, higher modes, the condition $\varepsilon << 1$, which essentially compares mean-flow velocities to linear-wave phase velocities, becomes progressively more restrictive. Note also that $k \neq 0$ is required in the disturbance (1.5), to preserve the ordering $|\varepsilon\omega_1| << |\omega_0|$ for sufficiently small ε in the expansion (1.8).

At $O(\varepsilon)$, the equation resulting from (1.6) is,

$$\begin{aligned} \omega_0 \left[-K^2 P_1 + \frac{d}{dz}\left(\frac{f^2}{N^2}\frac{dP_1}{dz}\right)\right] - \beta k P_1 \quad = \quad & -(\omega_1 - k\tilde{U} - l\tilde{V})\left[-K^2 P_0 + \frac{d}{dz}\left(\frac{f^2}{N^2}\frac{dP_0}{dz}\right)'\right] \\ & - \left\{k\frac{d}{dz}\left(\frac{f^2}{N^2}\frac{d\tilde{U}}{dz}\right) + l\frac{d}{dz}\left(\frac{f^2}{N^2}\frac{d\tilde{V}}{dz}\right)\right\} P_0, \end{aligned} \tag{1.13}$$

and the boundary conditions are,

$$\frac{dP_1}{dz} = -\frac{1}{\omega_0}\left(k\frac{d\tilde{U}}{dz} + l\frac{d\tilde{V}}{dz}\right)P_0 \quad \text{at} \quad z = \{-H, 0\}. \tag{1.14}$$

The differential operators on the left-hand sides of (1.9) and (1.13) have the same form, but the first is self-adjoint and the second is not, because of the different boundary conditions. Thus, the Fredholm Alternative is not directly applicable to (1.13), but a solvability condition may still be derived by multiplying (1.13) by P_0 and integrating over depth z, using integration by parts to simplify the left-hand side. This requires that

$$\left[-(k\frac{d\tilde{U}}{dz}+l\frac{d\tilde{V}}{dz})\frac{f^2}{N^2}P_0^2\right]_{z=-H}^{z=0} = \int_{-H}^{0} P_0\left\{-(\omega_1 - k\tilde{U} - l\tilde{V})\left[-K^2P_0+\frac{d}{dz}\left(\frac{f^2}{N^2}\frac{dP_0}{dz}\right)\right] - \left[k\frac{d}{dz}\left(\frac{f^2}{N^2}\frac{d\tilde{U}}{dz}\right)+l\frac{d}{dz}\left(\frac{f^2}{N^2}\frac{d\tilde{V}}{dz}\right)\right]P_0\right\}dz. \quad (1.15)$$

Thus, with P_0 normalized so that $\int_{-H}^{0}P_0^2\,dz = 1$,

$$\begin{aligned}\omega_1(K^2+\lambda^{-2}) &= \int_{-H}^{0}\left[(K^2+\lambda^{-2})(k\tilde{U}+l\tilde{V})+k\frac{d}{dz}\left(\frac{f^2}{N^2}\frac{d\tilde{U}}{dz}\right)+l\frac{d}{dz}\left(\frac{f^2}{N^2}\frac{d\tilde{V}}{dz}\right)\right]P_0^2\,dz \\ &\quad - \left[(k\frac{d\tilde{U}}{dz}+l\frac{d\tilde{V}}{dz})\frac{f^2}{N^2}P_0^2\right]_{z=-H}^{z=0} \\ &= \int_{-H}^{0}\left[(K^2+\lambda^{-2})(k\tilde{U}+l\tilde{V})P_0 - 2(k\frac{d\tilde{U}}{dz}+l\frac{d\tilde{V}}{dz})\frac{f^2}{N^2}\frac{dP_0}{dz}\right]P_0\,dz. \quad (1.16)\end{aligned}$$

It follows that, with an error of order ε^2,

$$\omega \approx \omega_0 + \varepsilon\omega_1 = \mathbf{k}\cdot\mathbf{U}_P + \Omega_*(k,l), \quad (1.17)$$

where $\mathbf{k} = (k,l)$,

$$\Omega_*(k,l) = -\frac{\beta_*^y k - \beta_*^x l}{K^2+\lambda^{-2}}, \quad (1.18)$$

the effective-β vector β_* is,

$$\beta_* = (\beta_*^x, \beta_*^y), \quad \beta_*^x = B, \quad \beta_*^y = \beta + A, \quad (1.19)$$

the mean-flow advection vector $\mathbf{U}_P$ is

$$\mathbf{U}_P = (U_P, V_P), \quad (1.20)$$

and the constants U_P, V_P, A, B are defined by,

$$\begin{aligned} U_P &= \int_{-H}^{0} U\, P_0^2\, dz, \\ V_P &= \int_{-H}^{0} V\, P_0^2\, dz. \\ A &= 2\int_{-H}^{0} \frac{f^2}{N^2}\frac{dU}{dz}\frac{dP_0}{dz} P_0\, dz, \\ B &= -2\int_{-H}^{0} \frac{f^2}{N^2}\frac{dV}{dz}\frac{dP_0}{dz} P_0\, dz. \end{aligned} \tag{1.21}$$

The dispersion relation (1.17) consists of a Doppler shift by the projected mean velocity components U_P and V_P, plus a dispersion function $\Omega_*(k,l)$ that has a form similar to the standard, rest-state dispersion relation (1.1), with the vector product $\hat{\mathbf{z}} \cdot [(k,l,0) \times (\beta_*^x, \beta_*^y, 0)] = \beta_*^y k - \beta_*^x l$ taking the place of the single term βk in (1.1). Thus, the $\mathbf{U}_P$ and β_* vectors, which are depth-independent constants, completely determine the speed and direction of propagation of linear waves on the weak, depth-dependent mean flow. For the case of long waves on a purely zonal mean flow ($K^2\lambda^2 << 1$ and $l = V = 0$), Killworth et al. (1997; their Section 3.a) and Colin de Verdière and Tailleux (2005; their Section 6) have previously obtained equivalents of A and U_P, by closely related calculations; in the case of Colin de Verdière and Tailleux (2005), the related result is exact in the long-wave limit, but the projection is on mean-flow modified modes, rather than on the rest-state modes, as here. Note also that, as for the higher vertical modes, the condition $\varepsilon << 1$, which relates mean-flow velocities to linear-wave phase speeds, becomes increasingly restrictive for progressively shorter waves, and can be expected to fail at sufficiently short wavelengths, for example, those at which baroclinic instability occurs.

Suppose that new coordinate axes (x', y') are chosen, which are rotated clockwise from

(x,y) by the angle $\theta = \arctan(\beta_*^x/\beta_*^y)$, so that

$$x' = (\beta_*^y x - \beta_*^x y)/|\beta_*|, \quad y' = (\beta_*^x x + \beta_*^y y)/|\beta_*|. \tag{1.22}$$

Then the corresponding rotated wavenumbers (k', l') must satisfy

$$k' = (\beta_*^y k - \beta_*^x l)/|\beta_*|, \quad l' = (\beta_*^x k + \beta_*^y l)/|\beta_*|, \tag{1.23}$$

in order that $k'x' + l'y' = kx + ly$. In these rotated coordinates, the modified dispersion function $\Omega_*(k,l)$ takes the standard form (1.1), with β replaced by $|\beta_*|$,

$$\Omega_*(k,l) = \Omega'_*(k',l') = -\frac{|\beta_*|\,k'}{K^2 + \lambda^{-2}}. \tag{1.24}$$

Thus, the wave-propagation component of the dispersion relation (1.17) for linear waves on the weak mean flow is precisely equivalent to the rest-state propagation, but pseudo-westward relative to the effective-β vector β_*, and with β replaced by the vector magnitude $|\beta_*|$.

2 Discussion

2.1 Energy transmission

Energy transmission for linear waves is determined by the group velocity $\mathbf{C}_g = (\partial\omega/\partial k, \partial\omega/\partial l)$. For the weak mean-flow dispersion relation (1.17),

$$\mathbf{C}_g = \mathbf{U}_P + \left(\frac{\partial \Omega_*}{\partial k}, \frac{\partial \Omega_*}{\partial l}\right). \tag{2.1}$$

Thus, the energy transmission consists of advection by the non-dispersive, projected mean-flow component $\mathbf{U}_P$, plus a dispersive component that, in view of (1.24), is precisely equivalent to the rest-state group velocity, but relative to the effective-β vector β_* rather than β itself.

For long waves, $K^2\lambda^2 << 1$, and $\Omega'_*(k',l') \approx -|\beta_*|\lambda^2 k'$. The long-wave group (and phase) velocity is then

$$\mathbf{C}_g^{lw} = (U_P - \beta_*^y\lambda^2, V_P + \beta_*^x\lambda^2), \tag{2.2}$$

which is a constant vector, so that the propagation is fully non-dispersive, with direction and magnitude determined by the combination of $\mathbf{U}_P$ and $\beta_*\lambda^2$.

2.2 The non-Doppler effect

If $dU/dz = dV/dz = 0$ at $z = \{-H, 0\}$, the mean flow may be projected onto the modes $P_0^{(n)}$ without loss of information at the boundaries, so that

$$U = \sum_{n=0}^{\infty} \hat{U}_n P_0^{(n)}, \quad V = \sum_{n=0}^{\infty} \hat{V}_n P_0^{(n)}. \tag{2.3}$$

Since then also

$$\frac{d}{dz}\left(\frac{f^2}{N^2}\frac{d\hat{U}_n}{dz}\right) = -\lambda_n^{-2}\hat{U}_n, \quad \frac{d}{dz}\left(\frac{f^2}{N^2}\frac{d\hat{V}_n}{dz}\right) = -\lambda_n^{-2}\hat{V}_n, \tag{2.4}$$

it follows from the first equality in (1.16) that the contribution from the first mean-flow mode $(U_1, V_1)P_0$ to the frequency correction ω_1 vanishes for long waves, that is, in the limit $K^2\lambda^2 \to 0$. Thus, the first-mode component of the weak mean flow has no effect on the linear long-wave propagation, because the effects of the corresponding potential vorticity gradient exactly cancel the advection. For arbitrary stratification and mean-flow vertical shear, this non-Doppler effect (Held, 1983; Killworth et al., 1997) is exact in the linear long-wave limit for the first mode. If the propagation of another mode is considered, instead of the first mode, it is easily seen that a similar non-Doppler effect obtains for the corresponding modal component of the mean flow.

2.3 Constant stratification and no shear at boundaries

Suppose $N = N_0 = constant$, so the n-th baroclinic mode is $P_0^{(n)} = (2/H)^{1/2}\cos(n\pi z/H)$, and $\lambda_n = N_0 H/(n\pi f)$. Suppose also that the mean flow satisfies $dU/dz = dV/dz = 0$ at $z = \{-H, 0\}$, and is expanded as

$$U = \sum_{n=0}^{\infty} \hat{U}_n P_0^{(n)} = \sum_{n=0}^{\infty} U_n \cos\frac{n\pi z}{H}, \quad V = \sum_{n=0}^{\infty} \hat{V}_n P_0^{(n)} = \sum_{n=0}^{\infty} V_n \cos\frac{n\pi z}{H}, \tag{2.5}$$

where $(U_n, V_n) = (2/H)^{1/2}(\hat{U}_n, \hat{V}_n)$. Then,

$$\mathbf{U}_P = (U_0, V_0) + \frac{1}{2}(U_2, V_2), \quad (A, B) = \lambda_2^{-2}(U_2, -V_2). \tag{2.6}$$

Thus, for constant N and a mean velocity profile that has no shear at the boundaries but is otherwise arbitrary, there are contributions to the weak mean-flow first-mode dispersion relation only from the barotropic and second-internal mode mean-flow components.

2.4 Zero-pressure bottom boundary conditions

Tailleux and McWilliams (2001; see also Killworth and Blundell, 2003) have suggested that linear wave theory may give dispersion relations more representative of observed low-frequency sea-surface height disturbance propagation if the standard bottom boundary condition $dP_0/dz = 0$ at $z = -H$ is replaced by $P_0 = 0$ at $z = -H$. This can be heuristically motivated as an effect of bottom roughness, which is assumed to reduced the disturbance amplitude near the bottom (see, e.g., Samelson, 1992), and results in a vertical mode structure that is roughly equivalent-barotropic, similar to some observed eddies. Results analogous to (1.17)-(1.21) can be readily derived for this case.

3 Examples

As examples, the β_*-vector and mean flow projection $\mathbf{U}_P$ derived above are computed here for climatological profiles of stratification and mean vertical shear at 30 °N latitude, 130 °W longitude, and at 35 °N latitude, 125 °W longitude (Fig. 1). For these profiles of buoyancy frequency $N(z)$ and the standard bottom boundary condition $dP/dz = 0$ at $z = -H$, the rest-state vertical modes have the familiar mid-latitude, open-ocean, vertically-intensified structure, with the first internal mode P_0 having a single zero crossings (Fig. 2). The projections (U_P, V_P) and the β_*-constants A and B were computed from these profiles directly from (1.21). For the computation of (U_P, V_P), the climatological profile of mean vertical shear was converted to mean absolute velocity by requiring zero depth-integrated velocity. The addition of a barotropic component to these velocity profiles would add the same velocity component to (U_P, V_P), since the vertical integral of P_0^2 is unity, and would have no effect on A and B.

For these profiles, the resulting values of A and B are positive, giving a β_*-vector that points northeastward, so that the corresponding pseudo-westward propagation would be northwestward (Table 1). Consistent with the non-Doppler effect, the signs of the contributions to the long-wave speed from the advective terms U_P and V_P, and from A and B, respectively, are opposite, so that the net long-wave propagation speeds (2.2) are smaller in magnitude than would be anticipated from the β_*-vector alone.

For comparison to the weak mean-flow results, the full solution of the linear long-wave problem with mean flow (1.6) was computed for $l = 0$; the corresponding first internal vertical modes are similar to, but measurably different from, the rest-state modes (Fig. 2). The values of the long-wave zonal phase or group velocity c^{lw} from the full long-wave solution are -0.0212 m s^{-1} and -0.0308 m s^{-1}, while the corresponding rest-state values are -0.0156 m s^{-1} and -0.0270 m s^{-1} (Table 1). In comparison, the weak mean-flow theory gives long-wave zonal propagation velocities of -0.0207 m s^{-1} and -0.0304 m s^{-1} for these two profiles, respectively, so the approximation appears to be quite accurate, at least for these examples and in the long-wave limit (Table 1).

Alternatively, the deviation $\delta\beta_{eff}$ from β of the weak mean-flow 'total effective-β' quantity $\beta_*^y - U_P/\lambda^2$, $\delta\beta_{eff} = \beta_*^y - U_P/\lambda^2 - \beta = A - U_P/\lambda^2$, may be compared with the equivalent quantity $-c^{lw}/\lambda^2 - \beta$ from the full long-wave solution. This comparison was done for sets of states constructed from each of the two example profiles, for which N^2 was held fixed while (U,V) was multiplied by a factor γ, where γ varied between 0 and 1. The results show the convergence of the weak mean-flow estimate toward the full long-wave result as $\gamma \to 0$ (Fig. 3).

The northwestward propagation indicated by the weak-mean flow analysis for the two example profiles is not consistent with indications of a tendency for southwestward propagation in this region of eddies tracked from satellite measurements of sea surface height (Schlax and Chelton, 2008). However, many other effects, including nonlinearities that are neglected here, may influence the meridional propagation of these eddies. If the propagation at the deformation-scale is computed instead, from (1.18)-(1.21) with $K^2 = \lambda^{-2}$, a southwestward phase propagation is obtained, due to the stronger influence of V_P relative to β_*^x, but it is not clear how or why this should be related to the observed eddy propagation. It is nonetheless hoped that a more systematic, future comparison of the directional characteristics of eddy trajectories with effective-β vectors may yet yield useful insights. The present results should in any case give useful approximations for wave propagation under the linearity and weak mean-flow conditions for which the derivation formally holds.

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I first met Peter Killworth in the summer of 1983, when he was the principal lecturer at the Summer Study Program in Geophysical Fluid Dynamics at Woods Hole, and in the years thereafter I was one of many who benefited from numerous stimulating interactions with him. His personal and scientific presence in the community will be greatly missed. This research was supported by the National Aeronautics and Space Administration, Grant NNX08AR37G. I am grateful to an anonymous reviewer for pointing out an error in the original submission. M. Schlax provided the two climatological profiles and helpful discussion on numerical solution of the full long-wave zonal-flow problem.

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Table 1: Parameter values for the two example profiles in Fig. 1. The approximate long-wave zonal phase speed $U_P - \beta_*^y \lambda^2$ from the weak mean-flow analysis may be compared with the value c^{lw} from the direct numerical solution of the long-wave eigenvalue problem and the rest-state zonal phase speed $-\beta\lambda^2$.

		35 ^{o}N, 125^{o}W	30 ^{o}N, 130^{o}W
β	(m^{-1} s^{-1})	1.86 $\times 10^{-11}$	1.97 $\times 10^{-11}$
λ	(km)	28.9	37.0
-$\beta\lambda^2$	(m s^{-1})	-0.0156	-0.0270
U_P	(m s^{-1})	3.21 $\times 10^{-3}$	5.95 $\times 10^{-3}$
V_P	(m s^{-1})	-8.72 $\times 10^{-3}$	-6.96 $\times 10^{-3}$
A	(m^{-1} s^{-1})	9.88 $\times 10^{-12}$	6.83 $\times 10^{-12}$
B	(m^{-1} s^{-1})	2.00 $\times 10^{-11}$	6.32 $\times 10^{-12}$
$A\lambda^2$	(m s^{-1})	8.26 $\times 10^{-3}$	9.34 $\times 10^{-3}$
$B\lambda^2$	(m s^{-1})	-1.67 $\times 10^{-2}$	8.64 $\times 10^{-3}$
$U_P - \beta_*^y \lambda^2$	(m s^{-1})	-0.0207	-0.0304
$V_P + \beta_*^x \lambda^2$	(m s^{-1})	0.0080	0.0017
c^{lw}	(m s^{-1})	-0.0212	-0.0308

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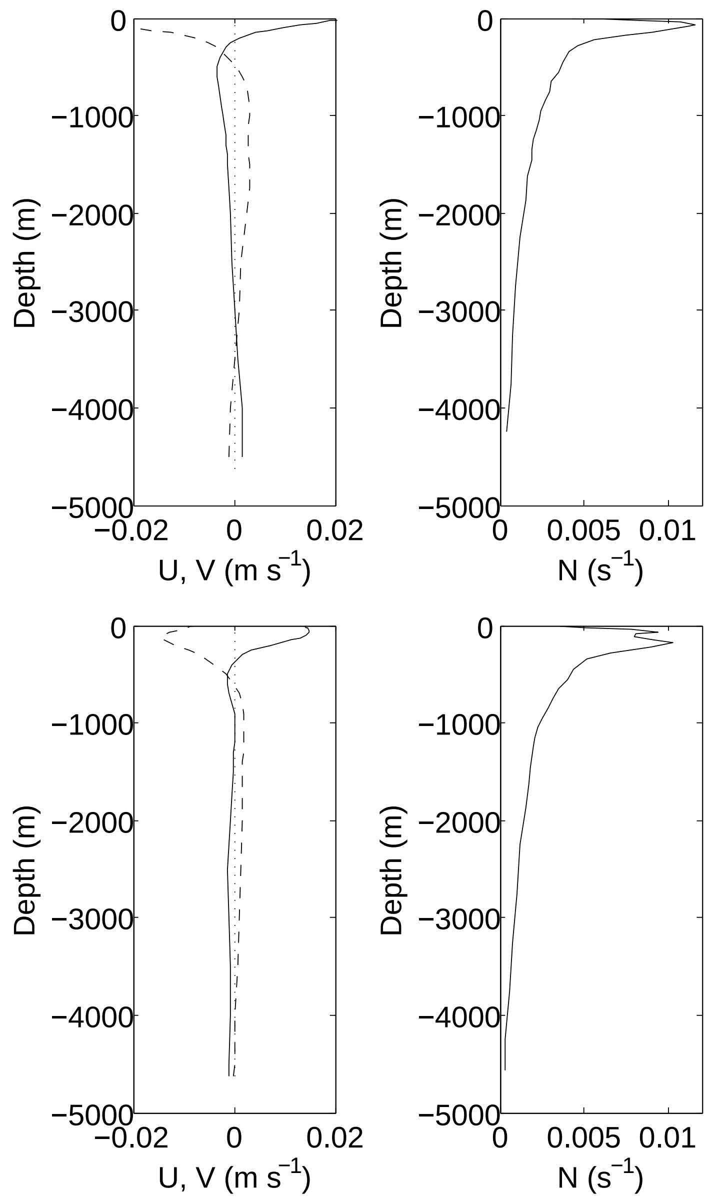

Figure 1: Mean flow profiles at (upper panels) 35 °N, 125 °W and (lower) 30 °N, 130 °W: velocity (left panels; m s^{-1}; U - solid line, V - dashed) and buoyancy frequency N (right; s^{-1}) vs. depth (m).

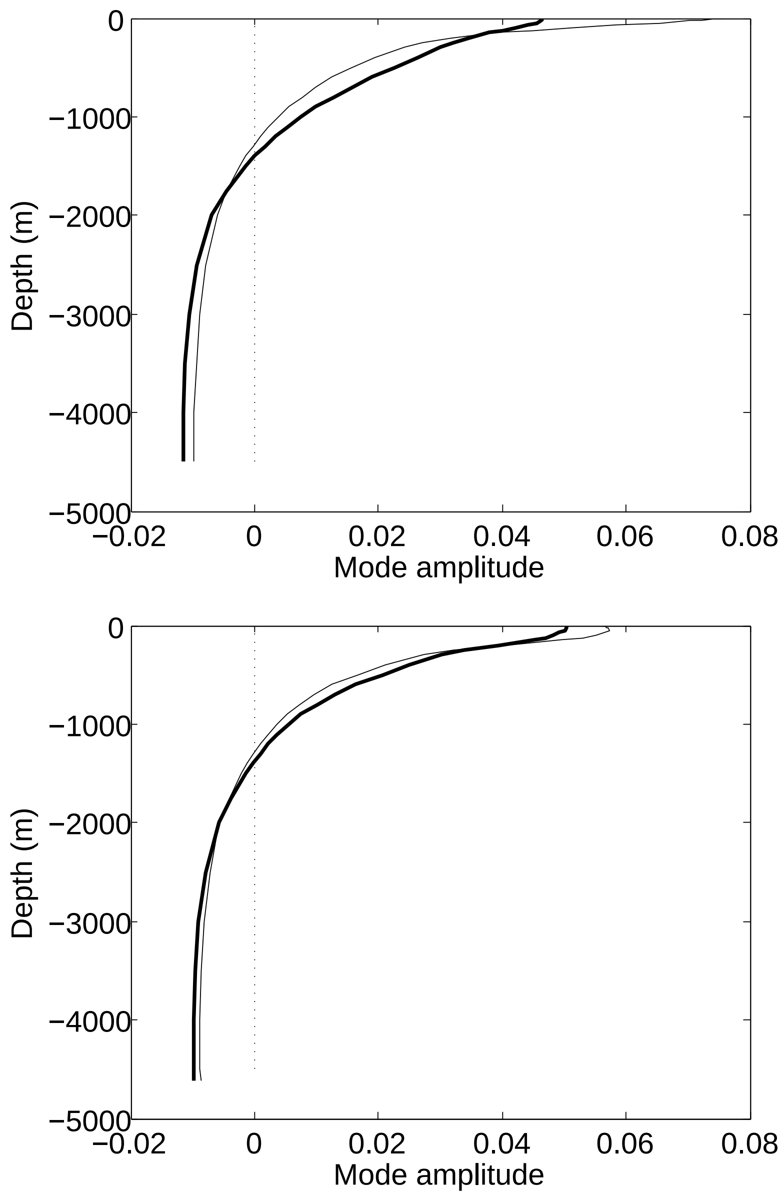

Figure 2: First vertical mode profiles $P_0 = P_0^{(1)}$ (thick solid lines) vs. depth (m) for the buoyancy frequency profiles in Fig. 1: (upper panel) 35 °N, 125 °W and (lower) 30 °N, 130 °W. The first vertical modes from direct numerical solution of the long-wave eigenvalue problem with purely zonal mean flow U, for the corresponding stratification and velocity profiles, are also shown (thin solid).

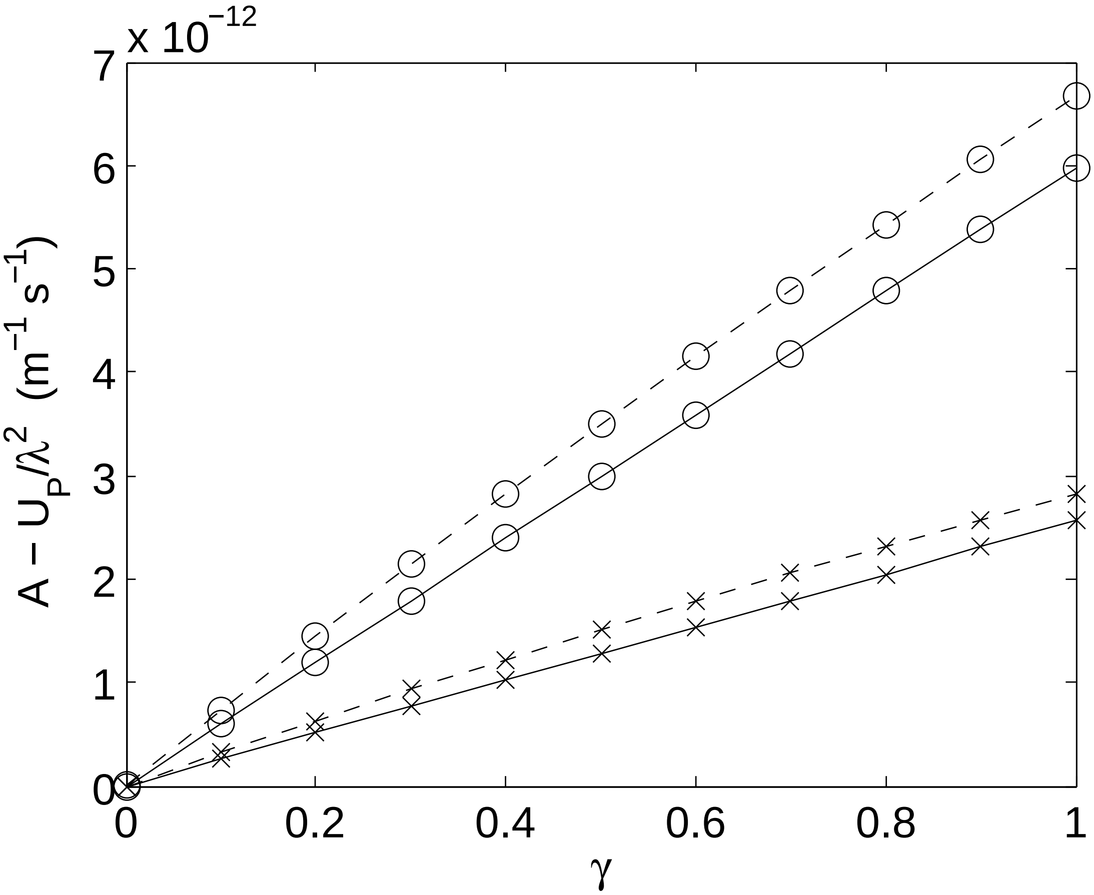

Figure 3: Weak mean-flow approximation (solid lines) to the departure $\delta\beta_{eff}$ of the 'total effective-β' quantity $\beta_*^y - U_P/\lambda^2$ from β vs. nondimensional parameter γ, for purely zonal mean flow γU, with U from the profiles in Fig. 1 ('○': 35^{o}N, 125 ^{o}W; '×': 30 ^{o}N, 130 ^{o}W. The corresponding equivalent quantity $-c^{lw}/\lambda^2 - \beta$ from direct solution of the long-wave eigenvalue problem with the same zonal mean flow is also shown (dashed lines).