Normal-Mode Instabilities of a Time-Dependent Coastal Upwelling Jet

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Abstract

The linear stability of a nearly time-periodic, nonlinear, coastal upwelling-downwelling circulation driven by a time-periodic wind stress is investigated using numerical methods. The near-periodic alongshore-uniform basic flow is obtained by forcing a primitive-equation numerical model of coastal ocean circulation with periodic wind stress. Disturbance growth on this near-periodic flow is explored in linear and nonlinear model simulations. Numerous growing normal modes are found in the linear analyses at alongshore scales between 4 and 24 km. These modes vary in cross shore structure and timing of maximum disturbance growth rate. One group of modes, in the 6.5-8.5 km alongshore scale range, bears strong resemblance to the ensemble average disturbance structures observed in perturbed nonlinear model simulations. These modes are of a mixed type, exhibiting both strong baroclinic and barotropic energy exchange mechanisms, with maximum disturbance growth occurring during the transition from upwelling favorable to downwelling favorable winds. Nonlinear disturbance growth is characterized by similar structures at these same scales, but with significant exchange of energy between disturbances at different alongshore scales, such that overall disturbance energy accumulates at the longest (domain) scales, and gradually propagates offshore mainly in the pycnocline over numerous forcing cycles.
1. Introduction

Although recent years have seen significant advances in our ability to understand and model coastal ocean flows, many important problems remain open. Progress has been made on the development and implementation of data-assimilating coastal ocean models (e.g., Oke et al. 2002), and there is a general appreciation that many coastal and eastern boundary current flows may be unstable (e.g., Barth 1989; Walstad et al. 1991). These instability processes represent a potentially important source of uncertainty in coastal ocean prediction models, but physical understanding of the associated processes and their role in coastal ocean systems remains limited. Experience from numerical weather prediction suggests that, given such uncertainties, ensemble forecasting will be a useful method to provide statistical predictive information in the coastal ocean.

Knowledge of the stability and transient growth properties of coastal ocean flows will be important for the development of ensemble coastal ocean forecasting systems and for the design of the next generation of coastal observing systems. In general, the practical problem of error growth in predictive models is closely related to the dynamical problem of instability and transient growth in time-dependent flows, and an understanding of both of these is important for the development of efficient ensemble generation methods. However, the linear instability problem for disturbances on flows with general time-dependence is complicated by the absence of the standard normal-mode framework, on which is founded most of our intuitive understanding of disturbance growth on unstable steady flows (e.g., Charney 1947; Eady 1949).
The study of disturbance growth on time-periodic basic flows offers an accessible middle ground: linear disturbances on time-dependent flows can still be studied within a normal-mode framework, but now the normal modes are themselves time-dependent. This approach has been taken, for example, by Samelson (2001, 2002), Samelson and Wolfe (2003), and Wolfe and Samelson (2006), who studied unstable periodic solutions and disturbance growth in a quasi-geostrophic channel model of nonlinear baroclinic waves. Strong et al. (1995), Chen et al. (1997), and Samelson and Tziperman (2001) have used related methods to study approximately periodic solutions of models of atmospheric variability and the El Nino/Southern Oscillation. The analysis of these unstable periodic solutions, or "unstable periodic orbits," is partly motivated by cycle expansion methods that have been developed for the study of low-dimensional chaotic systems (Artuso et al. 1990a,b). The extension of these methods to the study of spatio-temporal chaos in spatially extended systems is relatively recent (Christiansen et al. 1997; Zoldi and Greenside 1998). Because the fundamental complications introduced by time-dependence of the basic flow are present, but a complete analysis in terms of normal modes is still possible, this focus on periodic solutions offers a new and potentially valuable approach to study of the linear stability of time-dependent flows. The resulting normal modes are time-dependent, and have the structure of an exponential growth or decay multiplied by a temporally periodic function with complex spatio-temporal structure. These time-dependent normal modes, or Floquet vectors, are obtained by standard methods for linear differential equations with periodic coefficients (Coddington and Levinson 1955) and have proven useful for the analysis of parametric instabilities (Nayfeh and Mook 1979).
In many mid-latitude coastal regions, alongshore winds typically fluctuate on roughly 3-10 day timescales, due to synoptic-scale atmospheric variations. Motivated by this observation and more generally by the need to find a suitable starting point for the analysis of disturbance growth on time-dependent flow, we consider here the response of three-dimensional, primitive-equation numerical model of coastal ocean circulation to wind forcing that is periodic (sinusoidal) in time. For alongshore-uniform flow this response turns out itself to be essentially periodic. This cyclical, upwelling-downwelling response to oscillatory wind forcing is the basic flow, the stability of which to alongshore-varying perturbations is then investigated.

The goal of this study is to develop an understanding of disturbance growth in alongshore-uniform periodic coastal flows by examination of linear and nonlinear disturbance evolution about a near-periodic basic state. Temporal evolution of the basic state introduces the possibility that different perturbation scales and structures will be favored at different times. A nonlinear simulation of an unperturbed upwelling/downwelling cycle presents a near-periodic (but always alongshore uniform) flow structure. Here we examine the fastest growing, cyclic normal mode structures that develop about that flow and compare them to the structures that develop in perturbed nonlinear model simulations. Thus, the approach consists of 1) establishing a near-periodic basic flow cycle by running unperturbed (alongshore uniform) nonlinear model simulations, 2) determining the fastest growing normal modes about that flow through iterative solutions of the tangent linear model, and 3) analyzing an ensemble of nonlinear model simulations perturbed randomly at initialization, run over a number of forcing cycles.
2. Formulation

a. Numerical model

Linear and nonlinear numerical simulations were conducted with a hydrostatic, primitive-equation model (Shchepetkin and McWilliams 2003; Moore et al. 2004) in an idealized coastal ocean domain. The model configuration followed that used in Durski et al. (2007) but with higher spatial resolution in a shorter periodic channel. Domains of 150 km width, of different alongchannel lengths were used. A Cartesian coordinate system is used with $x$ aligned along-channel and $y$ aligned across-channel. The bathymetry is specified as an along-shore uniform, smoothed approximation to a typical across shore depth profile off the coast of Oregon, with the bottom depth increasing from 6 m at the coast (at $y = 0$ km) to 300 m at the offshore wall ($y = -150$ km). Bottom slopes nearshore are realistic but offshore the bathymetry flattens at approximately 300 m, a significantly shallower depth than is actually found off the Oregon coast. This preserves a highly resolved surface boundary layer across the entire domain, which helps avoid the development of spurious instabilities offshore related to errors introduced by the sigma-coordinate approximation over sloping bathymetry. The horizontal grid resolution is 333 m in the alongshore direction. In the cross shore direction, stretched grid spacing is used with approximately uniform 200 m resolution inshore of $y = -50$ km, expanding to 5-km resolution at the offshore boundary ($y = -150$ km). This high horizontal resolution is necessary to resolve the fast growing instability structures on this flow. Forty-five vertical sigma levels were employed. Potential density $\rho$ is used in place of
temperature and salinity, presupposing a linear equation of state. Other dependent variables include along-channel and across-channel velocities $u$ and $v$, respectively, vertical velocity $w$, and sea-surface height $\zeta$. For the simulations discussed here, the model is forced with surface wind stress only, and no surface heat or salinity fluxes were applied.

b. Tangent linear model

If the nonlinear primitive equation model (NLM) is expressed symbolically as a single evolution equation,

$$\frac{\partial S}{\partial t} = N(S),$$  \hspace{2cm} \text{(2.1)}

where $S$ is the model state vector and $N$ is the non-linear model operator, then the associated tangent linear model (TLM) may be obtained by a first order Taylor expansion of $N$ about a nonlinear solution $S_0(t)$,

$$\frac{\partial s}{\partial t} = \left. \frac{\partial N(S)}{\partial S} \right|_{S_0(t)} s = Ls.$$  \hspace{2cm} \text{(2.2)}

where $s$ represents the linear perturbation and $L$ is a linear operator. Note that the basic solution $S_0(t)$ may be time-dependent. For sufficiently small perturbations to the basic flow $S_0(t)$, the TLM will give an accurate approximation to the evolution of the full NLM.

The domain configuration and model settings for the TLM (linearized advection scheme, boundary conditions, etc.) were all identical to those in the nonlinear setup. The basic flow used in all the TLM calculations is the near-periodic flow described below (Section 3), approximated by linear interpolation between instantaneous states of the nonlinear model.
For this study, the basic state fields were interpolated between states that were 90 minutes (36 internal time steps) apart. Sensitivity tests revealed this to be the optimal interval for retaining solution quality while minimizing storage requirements. The vertical viscosity and diffusivity coefficients determined from the nonlinear basic flow (using Mellor and Yamada level 2.5, [1982]) were used in the TLM. The terms arising from the linearization of the flow-dependent mixing coefficients were set equal to zero.

c. Linear and Nonlinear experiments

Linear normal modes (Section 4) of the time-dependent basic flow were obtained within the ROMS framework using the TLM (Moore et al. 2004) and ARPACK (Lehoucq et al. 1997). This package solves for a few eigenmodes of a system iteratively, by integrating the TLM forward in time and re-initializing it with a new disturbance structure predicated on the output of the previous integration, until convergence to the requested number of (usually fastest growing) modes is reached.

Nonlinear disturbance growth (Section 5) is examined using an ensemble of 10 nonlinear model simulations. Each 35 day simulation, is initialized with a small random perturbation from the basic flow day 0 state. Disturbances of amplitude 10^{-3} kg/m^3 were added to the initial density field only. Here perturbations are defined as deviations from the alongshore average such that:

\[ \rho = \bar{\rho} + \rho', \]  

(2.3)
\[ \overline{\rho}' = 0, \quad (2.4) \]

where the overbar denotes the alongshore average of a field,

\[ \overline{\rho} = \frac{1}{L_x} \int_0^{L_x} \rho \, dx, \quad (2.5) \]

with \( L_x \) the length of the domain in \( x \).

3. Basic flow: upwelling-downwelling cycle

The basic near-periodic flow is obtained by integrating the nonlinear model under the influence of a sinusoidally varying wind stress from a state of rest until a near-periodic (in time) condition is reached. The initial vertical density profile is horizontally uniform and based on summer field observations off the Oregon Coast. The oscillatory wind stress is purely alongshore with a 5-day period. It is ramped up linearly over 50 days to a maximum peak amplitude of 0.1 N m\(^{-2}\). The model is then integrated an additional 165 days to reach a near periodic state. The end time of this 215-day spin-up will be referred to here and below as Day 0 of the basic cycle.

Since the domain, initial conditions and forcing were uniform alongshore, the circulation for this near-periodic cycle is two-dimensional, varying only with cross-shore distance \( y \) and depth \( z \). The phase of the wind forcing is such that the wind velocity vanished at days 0, 2.5, and 5, maximum upwelling-favorable wind occurred at day 1.25, and maximum downwelling-favorable wind occurred at day 3.75 in each 5-day period.
The predominant circulation feature over the cycle is a southward alongshore jet that reaches 0.4 m s$^{-1}$ on day 3 before weakening and disappearing under the influence of downwelling winds (Figure 1). The reverse, northward flow is significantly weaker. It develops earliest at the coast at about day 2.6 and largely remains confined within 4 km of the shore before dissipating towards the end of the cycle. The evolution of the depth-averaged alongshore flow corroborates this description (Figure 2). Isopycnals associated with water depths deeper in the pycnocline offshore (50-60 m depth), curve upward to the surface in the active 15 km upwelling/downwelling region closest to shore, oscillating in offshore position over several kilometers with the forcing. Isopycnals associated with slightly shallower depths offshore, dip downward closer to shore reaching a minimum at about the location of the base of the upwelling jet. In a very thin bottom boundary layer isopycnals stretch shoreward during upwelling and recede to deeper waters during downwelling with about a half-day lag relative to the forcing. The volume-integrated kinetic energy of the flow fluctuates during the cycle (Figure 3). Maximum kinetic energy coincides with the time of maximum velocity of the upwelling jet, or 'peak upwelling,' which occurs approximately $t=2.3$ days, shortly before the upwelling-favorable winds have fully relaxed.

This solution is largely independent of whether the initial 215-day integration, which starts from a resting state with level isopycnals, begins during the upwelling- or downwelling-favorable phase of the imposed wind forcing. The solution is not exactly periodic, as a slow evolution of the density field is evident on a longer time scale, and the volume integrated kinetic energy ($KE$) has a slight mean tendency (decreasing). The change in volume integrated...
The asymmetries in the speed and direction of the alongshore velocity during the up-welling and downwelling phases of the oscillation (Figure 1) are consistent with previous studies which have demonstrated marked differences, associated with nonlinear advection and mixing processes, between the finite-amplitude upwelling and downwelling responses (Allen et al. 1995; Allen and Newberger 1996; Kuebel-Cervantes et al. 2004). While oscillatory solutions of coastal ocean models are of interest in themselves, it is the properties and mechanisms of disturbance growth on this time-dependent flow that are of primary interest here. Disturbance growth on the cycle is studied with linear and nonlinear model simulations, as described in Sections 4 and 5. Early in this investigation the linear modes that develop on instantaneous snapshots of the periodic flow and time averages of the periodic flow were examined. These modes were found to bear little resemblance to the linear disturbance growth on the cycle and thus are not discussed further here.

4. Time-dependent linear normal modes

For the linear analysis, the TLM is used to calculate the fastest growing linear disturbances about the near-periodic two-dimensional basic flow. Since the basic flow is nearly periodic, the resulting disturbances can be identified as Floquet vectors, the fundamental normal-mode solutions to linear differential equations with periodic coefficients (e.g., Coddington and Levinson
These modes typically have the form

\[ \phi(x, y, z, t) = e^{\lambda t} \Phi(x, y, z, t), \]  
(4.1)

where \( \lambda \) is the Floquet exponent or growth rate, and \( \Phi \) is the time- and space-dependent structure function, which has temporal period equal to \( T \) or \( 2T \), where \( T \) is the period of the basic flow. That is, \( \Phi \) satisfies

\[ \Phi(x, y, z, t + T) = \Phi(x, y, z, t) \quad \text{or} \quad \Phi(x, y, z, t + T) = -\Phi(x, y, z, t). \]  
(4.2)

If \( \lambda > 0 \), the mode \( \phi \) grows exponentially, and the basic cycle is linearly unstable. If \( \lambda < 0 \), the mode \( \phi \) decays exponentially, and the basic cycle is linearly stable to disturbances of the form \( \phi \). If \( \lambda = 0 \), the mode \( \phi \) is neutral. Note that for time-periodic basic flows, there will always be at least one neutral mode, which is proportional to the time-derivative of the basic flow. The exponent \( \lambda \) may also be complex, with non-zero imaginary part; in that case, the stability is determined by the sign of the real part of \( \lambda \), while the imaginary part alters the period of the mode \( \phi \), so that it no longer corresponds to \( T \) or \( 2T \). In general, some other possibilities exist, if the system is degenerate (e.g., Coddington and Levinson 1955). From a numerical point of view, there will be as many Floquet modes as there are degrees of freedom in the model. Only the first few leading modes were computed here.

Two different numerical configurations for the state vector were used to solve for Floquet eigenmodes of the cyclic basic flow. The first method gives a general picture of how the
alongshore scale of the fastest growing modes is distributed, while the second method gives details about multiple modes that develop at any specified alongshore wavelength. Solving for many modes at many scales simultaneously is computationally prohibitive due to both the size of the domain required to gain the spectral resolution that is desired and due to the limitations of the numerical eigenmode solver (Lehoucq et al. 1997). In both configurations, the state vector for the eigenvalue problem is associated with the four dependent-variable fields $u$, $v$, $\rho$ and $\zeta$. In the first configuration, the six overall fastest growing modes were sought for a 24 km alongshore-scale domain with 250 m alongshore resolution. Because these modes could occur with multiple wavelengths, representing any of the resolved alongshore scales, this is referred to as the MW case. In this case the state vector consisted of the dependent-variable values at each model grid point. Since the basic flow is uniform alongshore, the linear problem is separable in the alongshore direction, and the linear modes could be sought independently at each alongshore wavelength. The second configuration is designed to find the three fastest growing normal modes at each of a set of specified alongshore wavelengths. For these experiments the state vector consisted of the Fourier coefficients at the specified wavelength at each $y$-$z$ location on the grid (e.g. $u = A_u(y,z) \cos kx + B_u(y,z) \sin kx$ where $k$ is the wavenumber corresponding to the desired wavelength). In this case the eigenmode solver was restricted to finding modes at fixed wavelengths so this set of calculations is referred to as the FW case.

Two distinctly different types of physical mode were found in the MW analysis, distinguished by their cross-shore and vertical structure, and the associated disturbance dynamics. In some of the analysis to follow, disturbance structure is examined in terms of the amplitude
and phase $P_{\Phi}$ of the cyclic structure functions,

\[ A_{d}^{\Phi} = |\mathcal{F}(\Phi_{d})[\kappa]| = (|\mathcal{F}_{R}(\Phi_{d})[\kappa]|^2 + |\mathcal{F}_{I}(\Phi_{d})[\kappa]|^2)^{\frac{1}{2}} \]  

(4.3)

and

\[ P_{d}^{\phi} = \tan^{-1} \frac{\mathcal{F}_{I}(\Phi_{d})[\kappa]}{\mathcal{F}_{R}(\Phi_{d})[\kappa]} \]  

(4.4)

where $d$ can be any state variable ($u$, $v$, $\rho$ or $\zeta$), $\mathcal{F} = \mathcal{F}_{R} + i\mathcal{F}_{I}$ indicates the Fourier transform in $x$ at each $y$-$z$ location, $\mathcal{F}_{R}$ and $i\mathcal{F}_{I}$ are the real and imaginary parts of $\mathcal{F}$ respectively, $\kappa$ is the alongshore wavenumber of the mode and vertical bars indicate absolute value.

The fifth-fastest growing MW mode, which is also the fastest-growing mode with 8-km alongshore scale, is distinct from the other four modes found in the MW experiment. It develops offshore in the region of the upwelling jet and shows significant structural evolution over the cycle (Figure 4). This mode appears to have three discernible components: 1) a surface trapped disturbance in the top 25 m of the water column that grows between days 1 and 2.5, which forms very close to the coast, 2) a disturbance that grows in the pycnocline at the base of the upwelling jet between days 1 and 3, and 3) a transition period between days 3 and 5 when the intensity of the disturbance near the bottom increases and the feature appears to collapse shoreward in a thin disturbance structure extending continuously from the surface nearshore to 40 m depth, 7 km offshore. A complementary perspective can be obtained by examining $y$-$t$ sections of the disturbance at fixed vertical level and alongshore position. Between days 1 and 3 the disturbance can be seen to develop about the center of the
upwelling jet (Figure 5) but the disturbance amplitude offshore of the jet maximum attenuates rapidly thereafter. Between days 3 and 4, the disturbance advances shoreward onshore of the upwelling jet core, aligning with the center of the maximum downwelling velocities between days 4 and 5. The disturbance is principally located above the bottom boundary layer over the first 3 days of the cycle (Figure 6). But around day 3, shortly after wind reversal, the magnitude of the disturbance decreases in the upper part of the water column and elevates near-bottom. Interestingly coincident with this transition at about day 3, the depth-averaged across shore velocity perturbation also strongly increases in magnitude and across shore scale.

The phase propagation of the disturbance changes significantly over the cycle (Figure 7). During peak upwelling the disturbance propagates downstream with the upwelling jet at approximately 37 cm s⁻¹. By day 3.5 its phase is nearly stationary and reverses to a slow propagation of approximately -7 cm s⁻¹ for the rest of the cycle.

In the surface jet feature of this fifth MW mode, the disturbance phase varies horizontally across the jet (Figure 8). Between days 1.5 and 3.5, the disturbance structure centered at the pycnocline beneath the jet exhibits a phase that increases sharply upward and at an angle offshore. By days 3.5 to 4.5 in the cycle, during the decay stage of the disturbance, the phase becomes nearly coherent in a thin filament that slopes from near the surface to the bottom. This is reminiscent of the barotropization process that typically halts disturbance growth in nonlinear baroclinic life cycles. Thus, this disturbance may evidently be interpreted as a baroclinic instability of the time-varying basic flow, the growth of which is modulated by changes in the basic flow, but for which the growth over each cycle period dominates the decay, yielding an
exponentially growing normal mode.

The relevance of this fifth linear MW mode to oscillatory behavior found in the nonlinear ensemble simulations is discussed below. The first four of the six modes sought in the MW analysis have a different physical character (The sixth mode sought did not successfully converge). They describe intense disturbance growth in the very shallow nearshore region, centered near the time when the wind shifts from downwelling to upwelling favorable (Figure 9). In descending order by growth rate, these modes are found at alongshore scales of 4.8, 12, 6 and 24 km. They represent four of the fastest growing modes in the 24-km domain. Perhaps because of their small vertical and cross-shore scales, they equilibrate rapidly, and are largely irrelevant to the behavior of the nonlinear ensemble simulations. In this way, they are loosely analogous to the small-scale convective instabilities in large-scale numerical weather prediction models, that typically have faster linear growth rates than the synoptic-scale baroclinic instabilities, but equilibrate at smaller amplitude and are less relevant to the problem of disturbance growth on the synoptic scale.

For the FW case, the TLM is configured with 14 different domain lengths: 4, 6, 7, 7.5, 8, 8.5, 10, 12, 14, 16, 18, 20, 22 and 24 km. In all of these simulation, there were 16 grid points in the alongshore direction so that the wavelength of interest is equally well resolved in each case. The 42 Floquet eigenmodes attained in these 14 FW calculations show a complex relationship between scale and growth rate (Figure 10). The imaginary part of the eigenvalues of these modes all differ from zero but without any discernible pattern as a function of wavelength. The fastest growing modes at scales above 8.5 km and below 6.5 km
are characterized by the rapid nearshore disturbance growth around day 0 that is identified in five of the six 24-km MW modes (Figure 9). The second and third fastest growing modes at these scales tend to have a disturbance structure that is more similar to the fastest growing 8-km mode, the distinct MW baroclinic-instability mode described above that is centered on the upwelling jet region (Figure 4). The three fastest growing FW modes at the 8 km scale are all similar in structure.

There are small differences in the growth rates observed with MW and FW methods. These are related to the different state vectors being used by the solver to determine the eigen-modes in the two cases. The MW method finds approximations to modes across the spectrum whereas the FW method determines modal structures that are guaranteed to only contain a single alongshore wavelength. Consequently the MW method is prone to contamination where the solver converges to a nearly pure single wavelength pattern but with some small amplitude signal of a different wavelength mixed in. The differences in the growth rates found for modes at scales of 12, 6 and 8 and 24 km is related to this. In fact the similarity in growth rate between the two methods does more to corroborate that each method is determining a robust mode than to call it into question. Six modes were actually requested from the solver in the MW experiment but the growth rate of the nominally 4 km scale mode determined was significantly less in the MW experiment than in the FW experiment. In conducting the FW experiments, the solver could not converge to any modes at the 4 km scale with the standard 120 second model time step which was used for all other FW experiments so the time step was reduced to 90 seconds. The 120 second time step which was used in the MW experiment resulted in a 4 km
scale structure that was an under-resolved and somewhat contaminated representation of the actual 4 km mode.

It is interesting to note that the 3 most rapidly growing modes in the 6.5-8.5 km wavelength band are very similar in magnitude, while at larger and smaller scales this is not the case. Evidently, the wind-forced upwelling-downwelling cycle is most unstable to jet-scale disturbances with alongshore wavelengths in this band. These wavelengths are comparable to the cross-shore scale of the upwelling jet in the basic cycle (Figure 1), and to the greatest cross-shore extent of the 8-km MW mode (Figure 4). This general correspondence of along-jet and cross-jet scales is typical for baroclinic instability of isolated jets.

To diagnose the sources of the energy for the instability growth, alongshore average and perturbation kinetic energy equations are derived (e.g., Orlanski and Cox 1973; Durski and Allen 2005). To facilitate physical interpretation, the equations are expressed in Cartesian coordinates. The alongshore-average total kinetic energy equals the sum of the kinetic energy in the average terms and the alongshore-average of the kinetic energy in the perturbation, i.e.,

\[ KE = KE_m + KE_p, \] (4.5)

where

\[ KE = \frac{1}{2} \rho_o (u^2 + v^2) \] (4.6)

\[ KE_m = \frac{1}{2} \rho_o (\bar{u}^2 + \bar{v}^2), \] (4.7)

\[ KE_p = \frac{1}{2} \rho_o (u'^2 + v'^2). \] (4.8)
The equation for the volume integrated kinetic energy in the alongshore-averaged flow is

$$\langle KE_m \rangle_t = -\langle cke \rangle - g\langle \bar{w}' \bar{\rho} \rangle + \langle KF \rangle + \langle KD \rangle. \quad (4.9)$$

The equation for the volume-integrated kinetic energy in the perturbations is

$$\langle KE_p \rangle_t = \langle cke \rangle + \langle cpe \rangle + \langle KF \rangle - \langle KF \rangle + \langle KD \rangle - \langle KD \rangle. \quad (4.10)$$

In these and the following equations the angle brackets $\langle \rangle$ represent volume integrals and the subscripts $t$, $x$ and $z$ denote partial differentiation. The kinetic energy transfer term $cke$ represents the sum,

$$cke = -\rho_o [\bar{u}' \bar{v}' + \bar{v}' \bar{w}' + \bar{u}' \bar{u}' + \bar{w}' \bar{w}']. \quad (4.11)$$

The transfer of potential energy to perturbation kinetic energy term $cpe$, is

$$cpe = -g\bar{w}'\bar{\rho}' \quad (4.12)$$

$KF$ represents the rate of change of kinetic energy associated with the surface and bottom stresses. $KD$ represents the dissipation due to vertical mixing processes. The constant reference density $\rho_o = 1000$ kg m$^{-3}$ and $g$ is the acceleration of gravity.

Some similarities and differences between growing linear modes can be observed by comparing the volume integrated $KE_p$ time series for each mode. Distinct similarities and
differences can be observed among the 42 modes determined in the FW experiments (Figure 11). Note that because these are growing modes, KEp is not periodic but reflects the net growth rate of the mode over the cycle. Between 7 and 8.5 km alongshore scales, the 12 largest normal modes determined all show peak values of $\langle KE_p \rangle$ at approximately mid-cycle. In these cases $\langle KE_p \rangle$ gradually increases over the upwelling favorable portion of the cycle then decreases over the downwelling portion. Similar shaped $\langle KE_p \rangle$ curves appear for modes at longer and shorter wavelengths, but never for the fastest growing modes. The fastest growing modes tend to show $\langle KE_p \rangle$ peaks at approximately day 1.5 and at the very end of the downwelling portion of the cycle, day 5. Many modes appear to be intermediate in structure between the other modes at the same scale and at the surrounding scales. For example the second fastest growing mode at the 8.5 km scale shows characteristics in $\langle KE_p \rangle$ which resemble that of the fastest and second fastest growing modes at the 10 km scale.

Examination of $\langle cke \rangle$ and $\langle cpe \rangle$ (where the angle brackets $\langle \rangle$ indicate volume integrals) for the 42 normal modes determined in the FW experiments reveal that many of the modes are dominated by kinetic energy conversion processes (Figure 12). In particular, rapid disturbance growth between days 0.5 and 2 and between days 4.5 and 5 is almost completely associated with large $\langle cke \rangle$. On the other hand, the strong peaks in $\langle KE_p \rangle$ that develop in some modes at the transition from upwelling to downwelling favorable winds at day 2.5 are associated with concurrently large values of $\langle cke \rangle$ and $\langle cpe \rangle$. The dominant peaks in the energy exchange term $\langle cke \rangle$ tend to coincide with those of $\langle KE_p \rangle$. Decreases in $\langle KE_p \rangle$ are generally associated with decreases in amplitude of $u'$ and/or $v'$ which contribute to $\langle cke \rangle$ likely causing this consistent
pattern between the two fields.

5. Nonlinear ensembles

The linear analysis described in Section 4 establishes that the near-periodic upwelling-downwelling solution identified in Section 3 is unstable to small disturbances. Thus, a fully nonlinear solution initially in a state close to the upwelling-downwelling solution will eventually depart from it, and develop different characteristics. The linear analysis is not sufficient to identify the characteristics of the resulting nonlinear solution. An ensemble of nonlinear simulations is constructed, in order to examine this behavior and its connection to the linear modes.

The nonlinear ensemble simulations are performed in a 48-km domain, initialized with small-amplitude perturbations superimposed on the two-dimensional basic flow at Day 0 of the cycle. As with the tangent linear model experiments, these experiments are performed with high resolution (approximately 250-m grid spacing in $x$ and $y$) to resolve adequately the disturbance structure that developed. Similar nonlinear simulations with lower cross-shore horizontal resolution show significantly weaker disturbance growth phenomena, indicating that the higher resolution is necessary. The ensemble consists of 10 nonlinear model simulations, each with a different initial perturbation. The perturbation amplitudes are specified to be approximately spectrally uniform over all resolved scales. Each simulation is integrated for 35 days, corresponding to 7 wind-forcing cycles.

After several cycles of the wind-forcing, the nonlinear simulations themselves ap-
proach solutions that have an oscillatory character. These modified upwelling-downwelling cycles have three-dimensional spatial structure, in addition to their temporal variability. There is significant variability amongst the ten ensemble members so in the discussion that follows ensemble averages will be considered as indicated by the notation \( \{ \} \). The ensemble-averaged volume-integrated perturbation kinetic energy, \( \{ \langle KE_p(k) \rangle \} \), computed at each alongshore wavelength \( k \) from the departures of the horizontal velocities from their alongshore means, increases rapidly over the first three cycles, and then approaches an approximately regular oscillation over the last four cycles (Figure 13). The initial rapid increase in \( \{ \langle KE_p \rangle \} \) is dominated by the growth of disturbances at the 7-8 km scale. During later cycles, the disturbance energy at these scales continues to increase rapidly during the acceleration of the upwelling jet, but also decreases strongly during the downwelling phase. Perturbation kinetic energy accumulates more at the domain scale than at any of the shorter wavelengths. The pattern suggested by this behavior in \( \{ \langle KE_p \rangle \} \) in the nonlinear model is that disturbances grow rapidly at the 7-8 km scale, and that even at saturation it is these scales which exhibit strong intermittent growth. However the kinetic energy generated at these scales evidently is taken up by longer scales over the course of each cycle. The initial disturbance scales that develop most rapidly and the tendency toward larger scales are both reminiscent of the evolution of frontal disturbances found in similar nonlinear simulations with steady wind forcing and time-varying forcing studied previously (Durski and Allen 2005; Durski et al. 2007). In Durski and Allen the 8 to 10 km scale disturbances that developed on the evolving upwelling front were identified as baroclinic instabilities.
Summed over all scales the largest deviations from the alongshore average develop in the pycnocline. However, at specific alongshore scales disturbance structure varies. Around the 8 km scale, the largest amplitude deviations from the alongshore average develop both in the upwelling jet and at the pycnocline beneath the jet (Figure 14). Here, the amplitude of the nonlinear disturbance in density at a particular alongshore scale is defined as

\[ A_{pl}^{nle}(\kappa) = \{ |\mathcal{F}(\rho^f)[\kappa]| \} \] (5.1)

analogous to (4.3) but where \{\} denote an ensemble average over the 10 simulations. The two regions of high disturbance amplitude are contiguous during some portions of the cycle and disjoint at other times. The cycle in disturbance structure at this scale persists for the full 35 days of the nonlinear model simulations (Figure 15) and a similar pattern is found at lower amplitude at shorter scales. The disturbance structures at scales greater than 8 km do not exhibit the same structure in the upper 30 m of the water column in the upwelling jet (Figure 16). However at all scales energy accumulates in the pycnocline and spreads farther offshore with each cycle (Figure 17).

The modes in roughly the 7 to 8.5 km band bear strong similarity to the persistent disturbance structure exhibited in the ensemble of nonlinear model experiments. They differ from the nonlinear structure mainly in that they have a more limited offshore extent in the pycnocline. Leading modes at longer and shorter wavelengths show much weaker resemblance to structures in the nonlinear model. As an additional test, nonlinear simulations were initialized with each of the 6 modes from the MW experiment individually as perturbations about the
2D initial state. These experiments showed that the nonlinear model failed to amplify modes 1, 2, 3, 4, and 6 for more than a day or two when initialized at very small magnitude, while the mode 5 (8 km) structure amplified in the case where it is initially present, and emerged in the cases where it is not. The particular relevance of this modal structure in the sustained nonlinear evolution may lay in the fact that it is associated with strong baroclinic energy exchange in addition to barotropic exchange. The ensemble average energy exchange terms for the nonlinear ensemble simulations suggest that, after the initial adjustment, baroclinic energy conversion dominates the oscillation dynamics over a broad range of scales (Figure 18). Another factor which may contribute to the prevalence of similar structures in the nonlinear model is that there are multiple relatively rapidly growing linear modes at the 7-8.5 km scales (Figure 10). \( \langle c_k \rangle \) and \( \langle c_p \rangle \) are almost never negative over the cycle (Figure 18). This requires that perturbation energy continue to accumulate in the system over repeated cycles or that energy growth be halted by the remaining terms associated with surface, bottom stress and vertical mixing in (4.9). The time series of \( \langle KE_p \rangle \) (Figure 13) and the steadily increasing amplitude of the disturbances in the pycnocline over numerous cycles (Figure 17) suggest that at the latest point in the nonlinear systems evolution examined here, the dissipative terms have slowed but not halted disturbance growth.

6. Summary

Disturbance growth in time-dependent flows representative of the Oregon coastal ocean has been studied by analysis of linear and nonlinear disturbance evolution about a near-periodic
basic flow, using a primitive-equation coastal ocean model with terrain-following coordinates. The near-periodic alongshore uniform basic flow is forced by oscillatory alongshore surface wind stress with 5-day period, and is essentially independent of the initial phase of the wind forcing. It consists of alternating upwelling and downwelling circulations, with a corresponding reversal of the alongshore flow. The upwelling and downwelling circulations are not symmetric, and the mean alongshore flow is dominated by the upwelling jet. The existence and general character of this periodically forced solution are consistent with a previous study motivated by observations in a different coastal region (Kuebel-Cervantes et al. 2004).

Temporal evolution of the basic state introduces the possibility that different perturbation scales and structures will be favored at different times. Normal modes on the cyclic basic flow, which take the form of Floquet eigenmodes, were computed using the tangent linear form of the primitive-equation model. The fastest-growing normal mode structures with scales comparable to that of the jet and centered on the jet region display many of the characteristics of baroclinic instability, modulated by the time-dependent fluctuations of the basic flow. These disturbance structures evolve considerably as the circulation transitions from upwelling to downwelling and consequently it is unlikely that similar flow structures could be attained from a steady state approximation of the basic flow. Faster-growing, smaller-scale normal modes are also found. Nonlinear simulations initialized close to the basic cycle approach an approximate, oscillatory, statistical equilibrium, some of the characteristics of which can be interpreted in terms of the dynamics of the baroclinic-instability Floquet modes.

The current generation of numerical models of coastal ocean circulation are sufficiently
accurate that meaningful dynamical insights can be obtained from the analysis of numerical simulations. Particularly for alongshore currents and coastal sea level, these models consistently show substantial hindcast skill. As practical coastal ocean forecasting based on dynamical forecasting develops, it is highly likely that ensemble forecasting methods will become important, because of large uncertainties in ocean initial conditions and in atmospheric forcing on ocean forecast time scales. Insight into the dynamics and characteristics of the instabilities of time-dependent flow is central to the rational construction of optimal ensemble forecast schemes. This study represents a modest first step toward that long-term goal.

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